A Study on the Modular Layout Principles of Sui-Tang Luoyang Based on Compositional Proportions

Weitang Shen
Fujian Agriculture and Forestry University
Yijing Chen
Fujian Agriculture and Forestry University
Zhenzhen Guo
Tongji University
Xinyi Wei
Fujian Agriculture and Forestry University
Yanhai Zhou
x9192246@gmail.com

Fujian Agriculture and Forestry University

Article

Keywords:

Posted Date: March 17th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-8072741/v1

Additional Declarations: No competing interests reported.

A Study on the Modular Layout Principles of Sui-Tang Luoyang Based on Compositional Proportions

Weitang Shen[1], Yijing Chen[2], Zhenzhen Guo[3], Xinyi Wei[4] and Yanhai Zhou*

Abstract

Sui-Tang Luoyang was an international metropolis during the 7th – 10th centuries, yet the layout principles that shaped its fundamental urban form have long remained undisclosed. This study takes the Danei as the benchmark and, based on the compositional proportions observed in the palace – imperial layout, combined with grid arrangement, identifies the modular relationships governing the entire capital. The research reveals that:(1) 24 *zhang* (70.56 m) served as the fundamental module not only for the Danei but also for the planning of the palace city, imperial city, and even the outer city.(2) Through the circle – square relationships embedded in compositional proportions such as 1:1, 5:7, 6:7 and 3:4, the entire capital achieved a symbolic expression of the cosmological concept "Round Heaven, Square Earth".Uncovering these modular layout principles will facilitate subsequent archaeological investigations and enhance the presentation and utilization of the heritage value of Sui-Tang Luoyang.

The form of a capital city is far more than a mere physical container; it represents the most concentrated spatial expression of national power order, cosmological concepts, and cultural ideals. For instance, the urban planning of Rome profoundly embodies the ancient Romans' pursuit of an ideal society and an eternal world[1]. From a broad, global civilizational perspective, the evolution of ancient Chinese capital city forms, particularly their ritualized planning tradition, constitutes a distinctive chapter in the worldwide history of capitals, attracting scholars from diverse fields to participate in its study. Current research on the morphology of ancient Chinese capitals can be broadly categorized into the following types:

a. General historical research. Scholars, placing capital cities within a Sino-Western comparative framework, summarize the universal morphological characteristics of ancient Chinese capitals: modeled on the ideal form described in the Kaogong Ji, exhibiting a strong political nature[2,3,4],possessing a pronounced central axis[5],and having an overall outline approximating a square, enclosed by walls[6].

b. Case studies of specific capital city forms. This includes research on Han Chang'an[7], Song Kaifeng[8], Yuan Dadu[9], and Ming-Qing Beijing[10], with the most extensive scholarly effort focused on Sui-Tang Chang'an. Scholars have thoroughly utilized increasingly rich archaeological materials and technologies such as GIS to conduct in-depth and detailed analyses of Sui-Tang Chang'an's spatial characteristics[11,12,13], morphological typology[14], grid system[15], land use characteristics[16], spatial patterns[17], transportation networks[18], and economic life[19].

*Corresponding author:Yanhai Zhou, Email: x9192246@gmail.com

[1] College of Landscape Architecture, Fujian Agriculture and Forestry University, Fuzhou, China

[2] College of Landscape Architecture, Fujian Agriculture and Forestry University, Fuzhou, China

[3] College of Architecture and Urban Planning, Tongji University, Shanghai, China

[4] College of Landscape Architecture, Fujian Agriculture and Forestry University, Fuzhou, China

c. In-depth research on influencing mechanisms. This type further explores the mechanisms through which factors like the natural environment[20,21], state politics[22], and national economy[23] influenced capital city form, focusing on the correspondence between capital city morphology and cosmological models[24,25,26].

The aforementioned three categories of research have largely addressed the issues of the formal presentation of ancient Chinese capitals and the explanation of their causative factors, significantly broadening academic horizons and greatly enriching research methodologies. However, they have all overlooked the question of the layout principles that directly shaped the planar forms of Chinese capitals. In fact, these layout principles are crucial. It is precisely the operation of this set of principles that ensured the continuity and stability of the planar morphology of ancient Chinese capitals, allowing them to form a self-contained system and stand out uniquely in the long river of world capital civilizations. The current state of research in this specific area is not optimistic, with only a few scholars like Fu Xinian and Wang Shusheng having made some progress[27,28]. This situation is largely attributable to the extreme scarcity of detailed drawings or textual records on capital planning from ancient China, which forces research on capital layout principles to rely heavily on archaeological materials. However, constrained by various objective factors such as overlying modern constructions on the sites, archaeological surveying and excavation work on capital city ruins can often only be conducted in limited areas. Consequently, it is difficult to obtain comprehensive and accurate archaeological data and measured plans for all zones of a capital city, making it challenging to abstract coherent layout principles from them.

Fortunately, due to the sustained efforts of archaeologists, a series of important discoveries at Sui-Tang Luoyang **(Fig. 1)** have been successively published, gradually bringing the city's overall scope, basic form, and regional layout—especially the palace city and imperial city located in the northwestern corner—back into clearer view. Notably, the compilation and publication of the *Sui-Tang Luoyang: Archaeological Excavation Report 1959-2001* (4 volumes) by the Institute of Archaeology, Chinese Academy of Social Sciences（CASS）, not only systematically presents nearly half a century of field archaeological work on Luoyang but also provides unified corrections to previous inconsistencies found in brief reports, reports, and data. Since 2001, excavation work on sites in the outer city such as Dingding Gate, Ningren Ward, and Zhengping Ward has progressed continuously, with related excavation reports or findings made public successively, providing more empirical evidence for exploring the gate regulations, road systems, and ward layouts of the outer city. Based on the availability of comprehensive and accurate archaeological data and measured plans for all major areas of Sui-Tang Luoyang, this paper selects it as the research object to investigate the planar layout principles employed at the initial planning and design stage of Sui-Tang Luoyang.

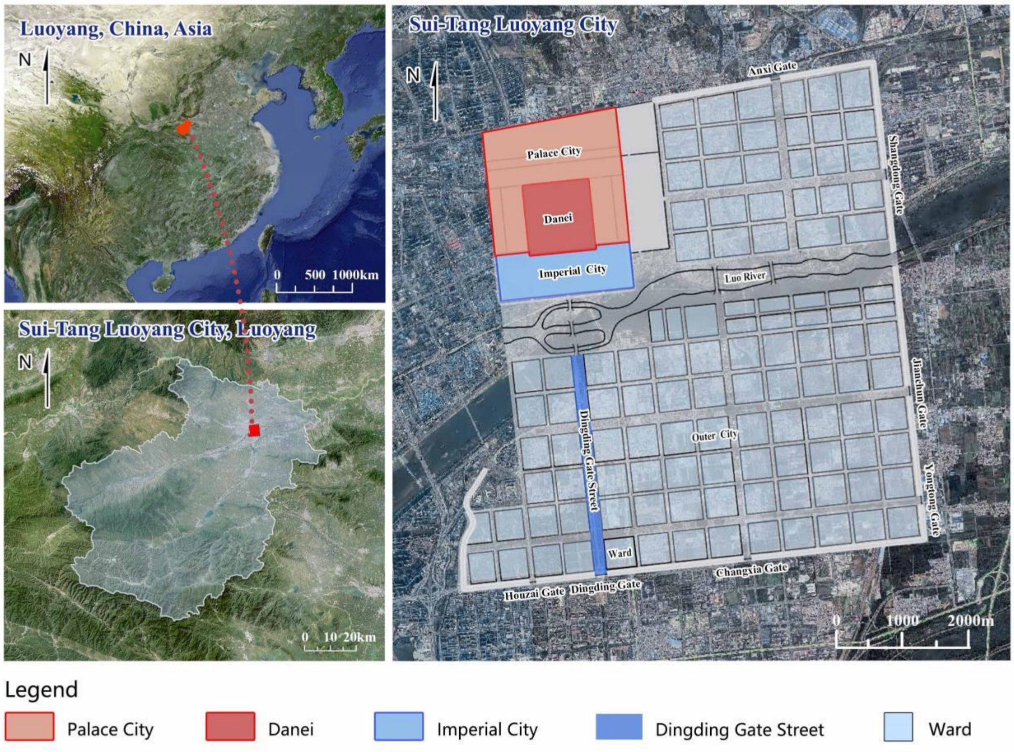

Fig. 1: Geographical Location Map of Sui-Tang Luoyang (Drawn by the author)

In recent years, facing various challenges brought by rapid urbanization, many scholars have focused on issues such as the spatial patterns of existing historical districts in ancient Luoyang[29] and urban sustainable development under the holistic protection of "Great Sites"[30]. Simultaneously, a small number of scholars continue to delve deeply into issues regarding the formation and transformation of Sui-Tang Luoyang's form[31]. Particularly commendable is Fu Xinian's exploration of the modular layout principles that shaped the form of Sui-Tang Luoyang . He proposed that the overall layout of Sui-Tang Luoyang was designed using the breadth and depth of the palace city as modules, with the palace city itself taking its core part—the square Danei —as the benchmark. Expanding the square Danei by 4 times yields the combined area of the imperial city and palace city. Dividing the square Danei into 4 parts results in each part equaling 1 ward. Later, Fu Xinian further deduced that the square Danei was designed and laid out using a module of 50 Tang-*zhang* (147 meters in modern measurement)[27].

Regrettably, as shown in **Fig. 2**， the modular unit derived by Fu Xinian applies only to the Danei and cannot be consistently applied throughout the palace city and imperial city, let alone establish consistency with the layout of the outer city and its internal streets and wards. In other words, the existence and applicability of this modular unit require further verification. However, Fu Xinian's methodology of combining calculation of archaeological measured data with geometric analysis is worthy of study and emulation. Furthermore, his approach of using the Danei as a benchmark to explore layout principles points the way toward deciphering the layout code of the entire city of Luoyang. This paper will continue to use the square Danei as the benchmark, first starting by searching for the smaller-scale modular unit used in the layout design of the palace and imperial cities, and then analyzing the relationship between the obtained modular unit and the large-scale layout design of the entire Sui-Tang Luoyang.

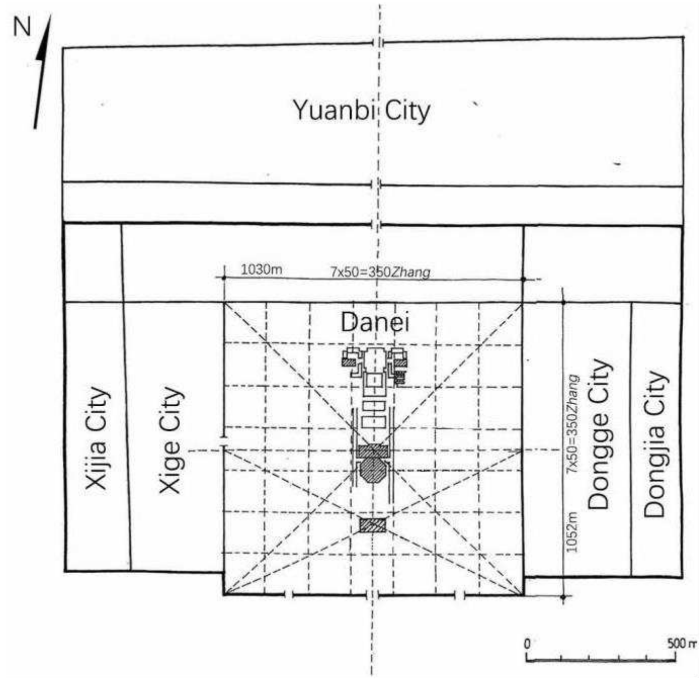

Fig. 2: Plan Analysis Diagram of the Da Nei in Sui-Tang Luoyang

Source:Fu Xinian, Research on the Methods of Ancient Chinese Urban Planning, Building Complex Layout and Architectural Design (Second Edition)

This paper hypothesizes that Yuwen Kai employed a basic module in the layout design process of Sui-Tang Luoyang that could simultaneously penetrate and connect the layouts of the palace city, imperial city, and outer city. If this modular unit is discovered through scientific deduction and verification, and if a clear spatial logic is established between all areas of the entire city of Luoyang via this modular unit, then the problem of the layout principles that directly shaped the form of Sui-Tang Luoyang is expected to be solved, thereby providing guidance for unraveling the mysteries of form-shaping in similar capital cities.

Methods

Research Object

Sui-Tang Luoyang, one of the eastern starting points of the UNESCO World Heritage "Silk Roads," exerted significant influence on cultural exchange between East and West from the 7th to the 10th centuries. Its construction commenced in the first year of the Daye era under Emperor Yang of Sui (605 AD) and it was successively used through the Tang, Five Dynasties, and Northern Song periods, lasting over 530 years. Known alongside Chang'an—another masterpiece by the same master planner, Yuwen Kai—it was called "Dongjing" (东京) in the Sui dynasty and "Dongdu" (东都) in the Tang dynasty. It served as the central node of the north-south Beijing-Hangzhou Grand Canal traversing East Asia. The fundamental layout of Luoyang remained essentially unchanged during the Sui and Tang dynasties, with updates confined to specific buildings in localized areas. The city's layout design holds a crucial position in the history of Chinese and even East Asian capital city construction, directly influencing subsequent Chinese urban development and serving as a template for city building in Japan's Asuka and Nara periods[32].

As a typical representative of traditional cities from the middle period of China's agrarian civilization, the Sui-Tang Luoyang site was listed among the first batch of 36 nationally protected "Great Sites" in 2005, officially included in the first list of National Archaeological Site Parks in 2010, and selected for the "100 Major Archaeological Discoveries of the Century" list in 2021.

Core Concept

The term "module" (abbreviated as "M" hereafter), also referred to as a "modular unit," is a core concept in ancient Chinese urban layout and architectural design. It denotes a fundamental dimensional unit repeatedly employed as a scale benchmark during planning and design. By combining integer multiples or simple fractions of this basic unit, designers determined the forms of urban districts, streets, wards, and the bay depths of building complexes, thereby establishing harmonious proportional relationships and a unified sense of order within complex spatial arrangements.

Source Materials

Unless otherwise specified, the data and base maps of the Sui-Tang Luoyang site used in this study are derived from two categories of materials:(1) *Sui-Tang Luoyang: 1959-2001 Archaeological Excavation Report* (4 volumes，Cultural Relics Press，2014) compiled by the Institute of Archaeology，CASS. (2) Archaeological reports of post-2001 excavations at sites such as the Dingding Gate and Ningren Ward published by the Institute of Archaeology，CASS ，and the Luoyang Municipal Institute of Cultural Relics and Archaeology， as well as information on archaeological progress at the Zhengping Ward site released from 2021 to the present.

Research Strategy

The primary objective of this paper is to identify and verify the basic module and its hierarchical levels used throughout the planning and layout of Sui-Tang Luoyang, based on the analysis of archaeological data. In this process, measured data will be converted into Sui-Tang period measurements as necessary, using the conversions 1 *zhang* = 10 *chi* = 2.94m, 1 *bu* = 5 *chi* = 1.47m, and 1 *li* = 180 *zhang* = 529.2m, in order to reconstruct the original numerical intentions.

The entire process is divided into two phases: The first phase seeks the module used in the layout of the centrally located palace city and imperial city. The second phase verifies the applicability of the module obtained in the first phase to the surrounding outer city. The former serves as the foundation, and the latter essentially repeats the steps of the former. The implementation steps for the first phase are detailed below:

Stage 1: Calculate Compositional Proportions

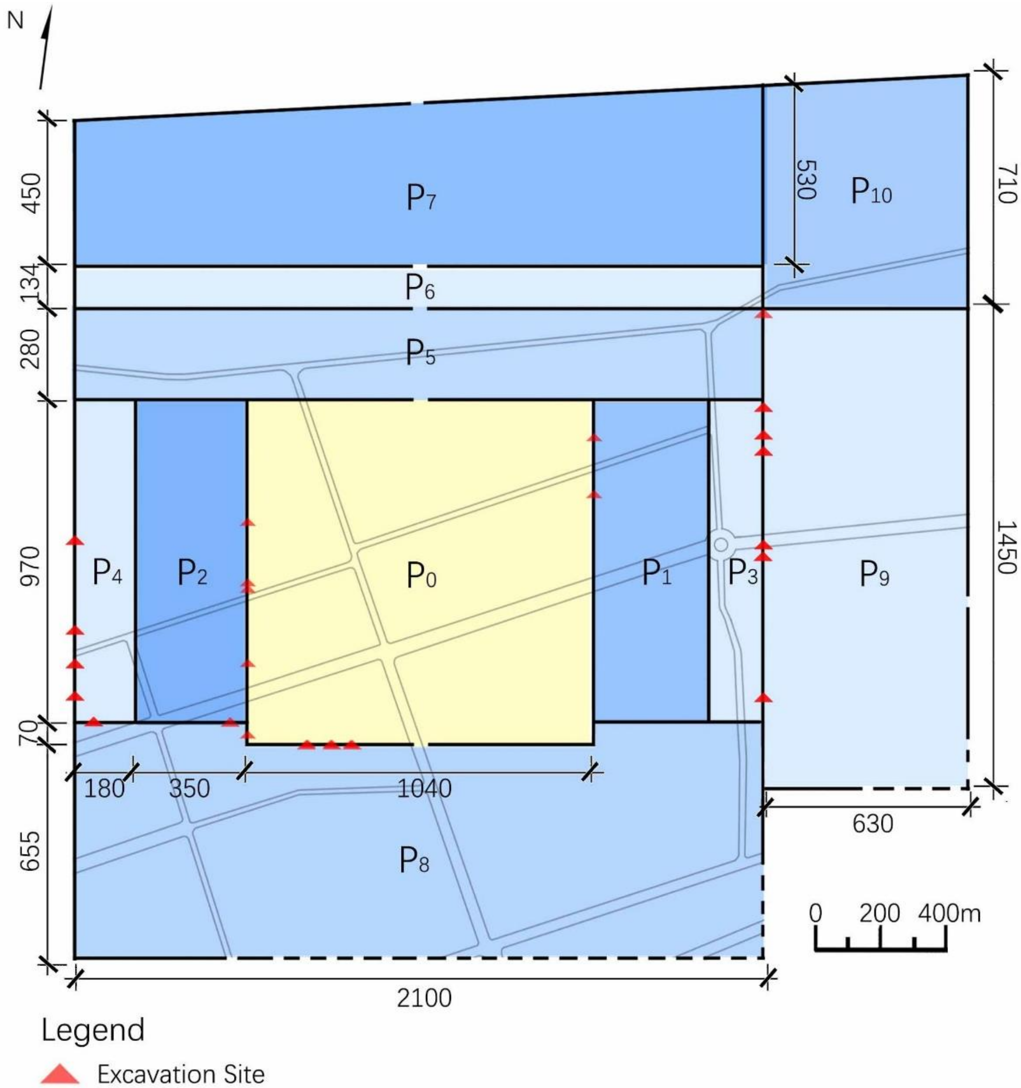

Fig.3: Plan Layout of Various precincts of the Palace/Imperial City of Sui-Tang Luoyang.

Precinct Code: P_0: Danei(大内)，P_1: Dongge City（东隔城），P_2: Xige City（西隔城），P_3: Dongjia City（东夹城）， P_4: Xijia City（西夹城），P_5: Xuanwu City（玄武城），P_6: Yaoyi City（曜仪城），P_7: Yuanbi City（圆璧城），P_8: Imperial City（皇城），P_9: Dong City（东城），P_{10}: Hanjiacang City（含嘉仓城）.

As shown in **Fig.3**, with the exception of the trapezoidal layout resulting from the slanted northern walls of Yuanbi City （圆璧城） and Hanjiacang City（含嘉仓城）, which resulted from topographical constraints, the layouts of all other precincts within the Palace city and Imperial city of the Sui-Tang Luoyang capital were composed of regular rectangles. The compositional proportions calculated in this study refer to the ratios between the measured dimensions of these rectangles and Danei (大内) in terms of east–west width (abbreviated as "W") and north–south depth (abbreviated as "D"), expressed as W_i/W_0 or D_i/D_0. Since Danei is square in layout, $W_0 = D_0$. The subscript i denotes the numbering of each precinct (abbreviated as "P"), with the baseline Danei numbered as P_0 and other precincts numbered from P_1 to P_{10}.

Preliminary calculation results are mostly decimals. To revert to the true intent during layout design, this paper converts the obtained ratios into approximate ideal integer ratios a_i/b_i, and then calculates the match rate between the survey data ratio and the ideal integer ratio. If the average match rate (μ) for each set of data is >97%, and the standard deviation (σ) is <2%, this paper posits that this integer ratio was used in the layout design.

Stage 2: Determine Modular Relationships

From the ideal integer ratios a_i/b_i obtained in the previous stage, there are denominators b_1, b_2, b_3, ..., b_{10}. These denominators must simultaneously satisfy the specific integer ratio relationships with the numerators a_1, a_2, a_3..., a_{10}. Therefore, the denominator should be the least common multiple of b_1, b_2, b_3, ..., b_{10}. From another perspective, since the integers b_1, b_2, b_3, ..., b_{10}, serving as denominators, correspond to the same square Danei's width and depth, the ratios a_i/b_i can be converted into fractions with a common denominator through finding equivalent fractions. The simplest common denominator is the Least Common Multiple LCM (b_1, b_2, b_3..., b_n). This LCM is essentially the number of parts into which the Danei's side length needs to be divided, and one of these parts is one modular unit. Then, dividing the Danei's width by the LCM gives the modular unit for the Danei:

$M = W_0 / LCM(b_1, b_2, b_3, ..., b_{10})$

From the previous stage, it is known that other precincts of the Palace and Imperial Cities have clear integer ratio relationships with the Danei in both width and depth. This study hypothesizes that they shared a certain module with the Danei. We continue by dividing the surveyed depth and width values of the various precincts of the Palace and Imperial Cities by this module, examining whether there is a clear integer multiple relationship – i.e., the match rate between the obtained value and the ideal integer value. Considering various factors that may cause errors, when the average match rate (μ) for each set of data is >97%, and the standard deviation (σ) is <2%, this paper concludes that the other precincts of the Palace and Imperial Cities used the same modular unit as the Danei.

Stage 3: Arrange the Modular Grid

First, pair the number of modular units in depth and width for each precinct of the Palace and Imperial Cities to determine the modular scale of different precincts. Using AutoCAD software, divide the surveyed plan of the Danei equally in the depth and width directions according to the obtained ideal multiples, resulting in a gridded survey plan of the Danei, where one grid cell corresponds to one modular unit. Then, using the same method, extend this grid cell on the survey plan until it covers the various precincts of the Palace and Imperial Cities, forming a grid layout diagram.

During this process, attention is paid to whether the grid lines basically coincide with the boundaries of the various precincts, and then whether the grid layout of each precinct matches its modular scale. If both hold true, it confirms that the same modular unit permeated the overall layout of the Palace and Imperial Cities.

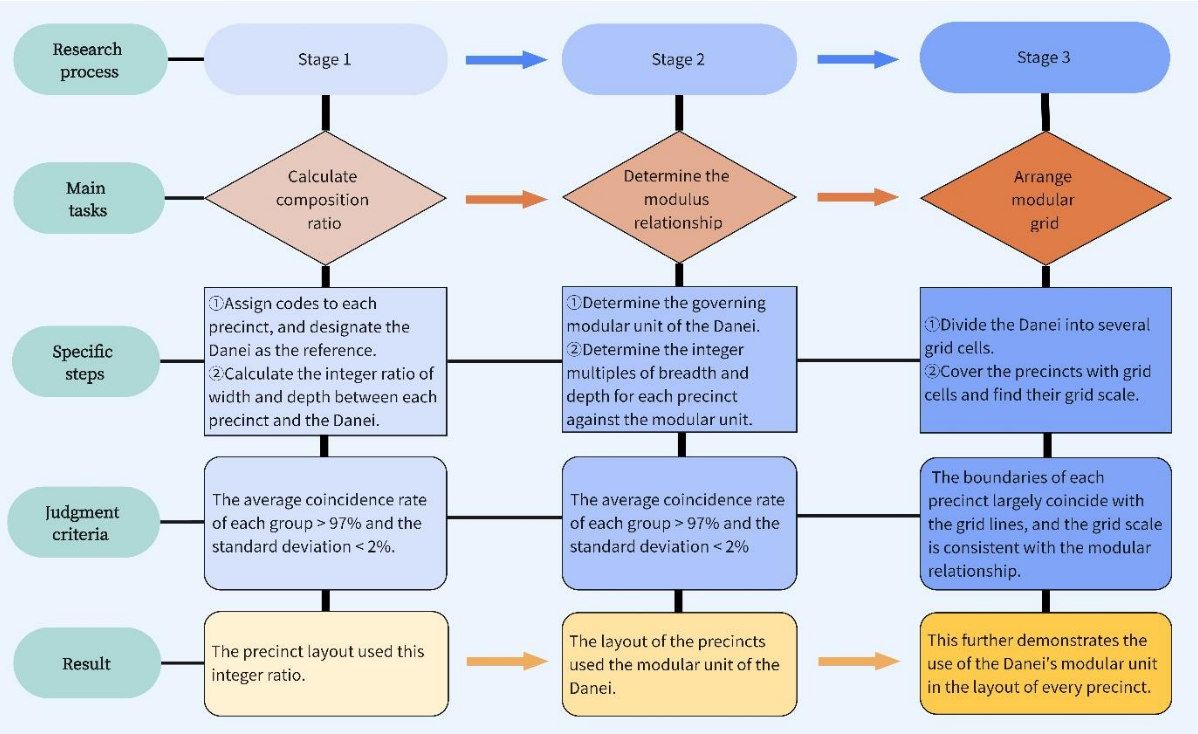

Fig.4: Technical Roadmap for Calculating the Modular Unit of the Palace/Imperial Cities

Results

Proportional Relationships between Various precincts of the Palace/Imperial City and the Danei in Width and Depth

Using the surveyed width W_0 and depth D_0 (1040m) of P_0 (Danei) as the benchmark， the calculation results are as follows:

Table 1: Integer Ratios of Palace/Imperial City precincts to the Danei in Width

Codes & precinct (P_i)	Surveyed Width (W_i)	Ratio to Danei Width (W_i/W_0)	Ideal Integer Ratio (a_i/b_i)	Integer Ratio Match Rate (%)
P_1: Dongge City	W_1: 350m	0.337	1/3	99.1
P_2: Xige City	W_2: 350m	0.337	1/3	99.1
P_3: Dongjia City	W_3: 180m	0.173	1/6	96.5
P_4: Xijia City	W_4: 180m	0.173	1/6	96.5
P_5: Xuanwu City	W_5: 2100m	2.019	2/1	99.1
P_6: Yaoyi City	W_6: 2100m	2.019	2/1	99.1
P_7: Yuanbi City	W_7: 2100m	2.019	2/1	99.1
P_8: Imperial City	W_8: 2100m	2.019	2/1	99.1
P_9: Dong City	W_9: 630m	0.606	3/5	99.0
P_{10}: Hanjiacang City	W_{10}: 630m	0.606	3/5	99.0

Table 2: Integer Ratios of Palace/Imperial City precincts to the Danei in Depth

Codes & precinct (P_i)	Surveyed Depth (D_i)	Ratio to Danei Depth (D_i/D_0)	Ideal Integer Ratio (a_i/b_i)	Integer Ratio Match Rate (%)
P_1: Dongge City	D_1: 970m	0.933	14/15	99.9
P_2: Xige City	D_2: 970m	0.933	14/15	99.9
P_3: Dongjia City	D_3: 970m	0.933	14/15	99.9
P_4: Xijia City	D_4: 970m	0.933	14/15	99.9
P_5: Xuanwu City	D_5: 280m	0.269	4/15	99.0
P_6: Yaoyi City	D_6: 134m	0.129	2/15	97.0
P_8: Imperial City	D_8: 725m	0.696	2/3	95.5
P_9: Dong City	D_9: 1450m	1.394	7/5	99.6

Note: P_7 and P_{10} have trapezoidal layouts and are therefore excluded from depth calculations.

As shown in **Tables 1 and 2**，we calculated the integer ratio match rates for the width and depth of various Palace/Imperial City precincts relative to the Danei. The 10 match rates in **Table 1** form one array，and the 8 match rates in **Table 2** form another. Placing these two arrays on either side of a single back-to-back stem-and-leaf plot yields the following:

Fig.5: Back-to-Back Stem-and-Leaf Plot of Width/Depth Integer Ratio Match Rates (Unit:%)

Left Leaf: Width Ratios	Stem	Right Leaf: Depth Ratios
	100	
0 0 1 1 1 1 1 1	99	0 6 9 9 9 9
	98	
	97	0
5 5	96	
	95	5

From the stem-and-leaf plot，the mean match rate for width integer ratios (μ) = 98.56%，standard deviation (σ) = 1.03%; the mean match rate for depth integer ratios (μ) = 98.84%，standard deviation (σ) = 1.68%. Comparison shows the means and standard deviations of the two arrays are very close. The data points are mostly clustered between 99.0% and 99.9%，rather than being continuously distributed. Specific analysis is as follows:

(1)The extremely high mean match rates indicate that the layout design of the Palace/Imperial City successfully applied the "ideal integer ratios" in construction practice. Such high means are by no means accidental; they strongly demonstrate that planners，represented by Yuwen Kai，possessed a clear sense of proportional design and superb surveying and construction technology，enabling precise layout implementation in large-scale capital construction.

(2)Against the benchmark of 98.56% and 98.84%，standard deviations fluctuating within 1.68% represent a relatively small amplitude. This suggests that while strictly adhering to the core integer ratio principle，planners might have made minor adjustments based on specific terrain，existing structures，or functional requirements.

(3)The non-normal，highly clustered distribution of both arrays is common in modular system research for urban planning and architectural design，consistent with the characteristic use of regular modular multiples in layout design.

The clear integer ratio relationships between the Danei and other precincts of the Palace/Imperial City in both the east-west and north-south directions suggest a potential modular relationship between them. One point is clear: the Danei held a core position in the layout design of the Palace/Imperial City，ensuring a sense of order and logic in their layout. This also confirms Fu Xinian's assertion that Yuwen Kai indeed designed the Palace/Imperial City layout using the square Danei's side length as the benchmark from the outset.

The Danei's Modular Unit and Its Relationship to the Palace and Imperial Cities

Section 3.1 obtained the ideal integer ratios for the width and depth of various Palace/Imperial City precincts relative to the Danei. Now we find the Least Common Multiple (LCM) of the denominators of these two sets of ideal integer ratios，i.e.，the number of parts into which the Danei's width and depth need to be divided:

Width direction: LCM (3，3，6，6，1，1，1，1，5，5) = 3 × 2 × 5 = 30
Depth direction: LCM (15，15，15，15，15，15，3，5) = 3 × 5 = 15

The number of parts for dividing the square Danei's side length: should it be 30 or 15 parts? The number of parts required for the width is twice that for the depth. Theoretically，the former number should also suit the latter. However，from the perspective of planning and construction convenience，the latter number is more feasible. Because the number of parts is inversely proportional to the modular unit size，and the Danei's 1040 meters is a relatively large scale requiring a larger modular unit，resulting in a smaller number of parts. A smaller number of parts is more convenient for the layout design and subsequent construction of the Danei and the even larger-scale Palace/Imperial City.

1040 ÷ 15 = 69.33m. That is，dividing each side of the Danei into 15 parts means 1 part is 1 module，equaling 69.33m. It is important to clarify that this 69.33m is the modular unit calculated from the survey data and is not necessarily equal to the module used during the original layout design. Numerous steps from initial layout design to final construction and field surveying can introduce errors，so the two are not the same number and certainly have some deviation. Therefore，it is necessary to convert 69.33m into Sui-Tang period measurements to find the "design module" used during planning(Table 3)

Table 3: Design Modules Corresponding to the Surveyed Module of 69.33m

Surveyed Module (m)	**Conversion Standard (m/*zhang*)**	**Conversion Result (*zhang*、*bu*)**	**Design Module (*zhang*、*bu*)**	**Design Module Length (m)**	**Surveyed Module Match Rate (%)**
69.33	2.94m/*zhang*	23.58 *zhang*	24 *zhang*	70.56	98.2%
69.33	1.47m/*bu*	47.16 *bu*	48 *bu*	35.28	98.2%

Converting 69.33m to Sui-Tang *zhang* or *bu* yields 23.58 *zhang* (47.16 *bu*), with a match rate of 98.2% to 24 *zhang*. Based on this, 24 *zhang* is preliminarily deduced as the module (M) used by Yuwen Kai in the planning and layout design. Twenty-four Tang *zhang* equals 70.56 meters in the modern metric system. A subtle point often overlooked is that the depth of the Annex and Separation Cities is recessed northward by about 70m compared to the Danei, leaving a depth of 970m. The surveyed 69.33m falls within the normal error range for 70.56m, with an error rate of only 1.2%. Ancient Chinese craftsmen generally used integers in planning and construction. The Danei's south wall protrudes one modular unit south compared to the eastern and western Separation and Annex Cities, and the remaining dimensions are also integer multiples of this modular unit. This situation occasionally occurs in ancient architectural layouts.

Using M = 70.56m (24 *zhang*) as the modular unit, the calculation results for the modular relationships of various Palace/Imperial City precincts are as follows:

Table 4: Modular Relationships for Width of Palace/Imperial City precincts

precinct Name	Surveyed Width (m)	Design Module Unit	Surveyed Module Multiple (M)	Design Module Multiple (M)	Surveyed Multiple Match Rate (%)
P_0: Danei	W_0: 1040	M	14.74M	15M	98.3
P_1: Dongge City	W_1: 350	M	4.96M	5M	99.2
P_2: Xige City	W_2: 350	M	4.96M	5M	99.2
P_3: Dongjia City	W_3: 180	M	2.55M	2.5M	98.0
P_4: Xijia City	W_4: 180	M	2.55M	2.5M	98.0
P_5: Xuanwu City	W_5: 2100	M	29.76M	30M	99.2
P_6: Yaoyi City	W_6: 2100	M	29.76M	30M	99.2
P_7: Yuanbi City	W_7: 2100	M	29.76M	30M	99.2
P_8: Imperial City	W_8: 2100	M	29.76M	30M	99.2
P_9: Dong City	W_9: 630	M	8.93M	9M	99.2
P_{10}: Hanjiacang City	W_{10}: 630	M	8.93M	9M	99.2

Note: Design Module Multiple represents the ideal multiple of the modular unit used in layout design.

Table 5: Modular Relationships for Depth of Palace/Imperial City precincts

precinct Name	Surveyed Depth (m)	Design Module Unit	Surveyed Module Multiple	Design Module Multiple	Surveyed Multiple Match Rate (%)
P_0: Danei	D_0: 1040	M	14.74M	15M	98.3
P_1: Dongge City	D_1: 970	M	13.75M	14M	98.2
P_2: Xige City	D_2: 970	M	13.75M	14M	98.2

precinct Name	Surveyed Depth (m)	Design Module Unit	Surveyed Module Multiple	Design Module Multiple	Surveyed Multiple Match Rate (%)
P_3: Dongjia City	D_3: 970	M	13.75M	14M	98.2
P_4: Xijia City	D_4: 970	M	13.75M	14M	98.2
P_5: Xuanwu City	D_5: 280	M	3.97M	4M	99.3
P_6: Yaoyi City	D_6: 134	M	1.90M	2M	95.0
P_8: Imperial City	D_8: 725	M	10.27M	10M	97.2
P_9: Dong City	D_9: 1450	M	20.55M	21M	97.9

Note: As in Table 2， P_7 and P_{10} are excluded from calculation.

Creating a back-to-back stem-and-leaf plot from the surveyed module multiple match rates in Tables 4 and 5:

Fig.6: Back-to-Back Stem-and-Leaf Plot of Width/Depth Surveyed Module Multiple Match Rates (Unit: %)

Left Leaf: Width Direction	Stem	Right Leaf: Depth Direction
	100	
2 2 2 2 2 2 2 2	99	3
3 0 0	98	2 2 2 2 3
	97	2 9
	96	
	95	0

The modular relationships of the various Palace/Imperial City precincts are analyzed as follows:

(1) The mean integer multiple match rate for width (μ) = 98.9%，with a standard deviation (σ) = 0.50%. The high mean indicates that，in the width direction，the surveyed dimensions of all precincts align very closely on average with integer multiples of the ideal modular unit (24 *zhang*)，nearing 99%. In other words，the width dimensions of each precinct almost perfectly conform to the integer multiple relationship based on the 24 *zhang* module，with minimal design or measurement error. The small standard deviation shows very little variation between the width match rates of different precincts，with data points highly concentrated around the mean.

(2) The mean integer multiple match rate for depth (μ) = 97.83%，with a standard deviation (σ) = 1.12%. The mean is slightly lower than for width but still represents a high match rate，indicating that depth dimensions also generally follow the modular relationship. The standard deviation is slightly higher than for width，primarily due to the smaller surveyed

depth value of Yaoyi City， which increases the overall fluctuation， while other precincts are mostly around 98%.

(3) The distribution of both arrays is also relatively concentrated rather than continuously varying， a characteristic that aligns well with the highly consistent execution of modular integer multiples in layout design.

In summary， the statistical results validate the applicability of the 24 *zhang* (70.56 m) modular unit in the layout design of the Palace and Imperial Cities， revealing inherent layout rules. This sufficiently demonstrates that Yuwen Kai's layout design involved meticulous calculation and a rigorous process.

Here two points require explanation:

First， The Symbolic Meaning of the Modular Relationships in the Danei.The Danei employed a modular unit of 24 *zhang*. Its overall width and depth were both designed using a scale of 15M (where M represents the 24 *zhang* module). The numbers 24 and 15 here correspond to constants in traditional Chinese calendrical astronomy. A year comprises 24 solar terms，and each solar term was traditionally considered to consist of 15 days. Multiplying these two numbers (24 × 15) yields 360， representing a complete celestial cycle， also conceptualized as "*one Yuan*" (一元). The seminal lexicon *Shuowen Jiezi*(《说文解字》) explicates these concepts at its outset: "The *One* (一) signifies the primordial commencement ，the *Dao*（道）established within the One， which brought forth the division of Heaven and Earth and gave form to all things." It further states，"*Yuan* signifies the beginning."[33] The classical commentary *Chunqiu Gongyang Zhuan* (《春秋公羊传》) articulates: "What is meant by *Yuan*? It is the inaugural year of a ruler."[34] By incorporating these two numbers into the design of the Danei， the architect Yuwen Kai symbolically represented the inauguration of a new political order. This symbolism also resonated with the mindset of Emperor Yang of the Sui dynasty，who，in the early years of his reign，perceived himself as the universal sovereign ruling over a unified empire.

This mode of thought， which draws analogies between measurements and numbers from calendrical astronomy， is a hallmark of the "Imitating Heaven and Modeling Earth" (象天法地) tradition prevalent in the planning of ancient Chinese capitals. According to the official history *Suishu*（《隋书》）， the chief planner Yuwen Kai hailed from an illustrious family and was described as "showing promise and capability from a young age... Kai was exceptionally studious， widely read in classical and historical texts， proficient in literary composition， and possessed of many talents and technical skills." He was later entrusted with directing several major state construction projects， including the capital cities Chang'an and Luoyang， as well as the Renshou Palace. In his efforts to reconstruct the Mingtang (明堂)， he conducted extensive research into classical sources and submitted the *Memorial on the Mingtang* (《明堂议表》). This memorial is thoroughly permeated with this form of numerical symbolism， containing statements such as: "The surrounding watercourse is 24 *zhang* wide， symbolizing the 24 Solar Terms"; "The twelve halls correspond to the Twelve Months"; and "The seventy-two windows correspond to the number of days associated with the cyclical movements of the *Wuxing*(五行)."[35]

Second，the issue of the 2.5 multiple for the width of the Dongjia City and Xijia City. This is a relatively common phenomenon in ancient construction practice，known as a "*half-unit*"(半数) or understood as a reduced module，often resulting from fine-tuning the module system based on practical needs. If the widths of the Dongjia and Xijia Cities are added together，they total 5 modular units，which is also an integer. This ensures a consistent modular relationship across the entire precinct while demonstrating the flexibility of the modular system in practical application.

Modular Scale and Grid Layout of the Palace/Imperial City

Pairing the design module multiples for width and depth from section 4.2 for each precinct of the Palace/Imperial City yields their modular scales as shown in the table below:

Table 6: Modular Scale Table for Palace/Imperial City precincts

precinct Name	Design Width Multiple	Design Depth Multiple	Design Modular Scale
P_0: Danei	15M	15M	15M × 15M
P_1: Dongge City	5M	14M	5M × 14M
P_2: Xige City	5M	14M	5M × 14M
P_3: Dongjia City	2.5M	14M	2.5M × 14M
P_4: Xijia City	2.5M	14M	2.5M × 14M
P_5: Xuanwu City	30M	4M	30M × 4M
P_6: Yaoyi City	30M	2M	30M × 2M
P_7: Yuanbi City	30M		
P_8: Imperial City	30M	10M	30M × 10M
P_9: Dong City	9M	21M	9M × 21M
P_{10}: Hanjiacang City	9M		

Note: M = 24 *zhang* = 70.56 m. As in Table 2， P_7 and P_{10} are excluded from depth calculations.

From this grid layout analysis diagram (**Fig.7**)，we can clearly see that the grid lines or centerlines basically coincide with the boundary lines of the surveyed plan of the Palace/Imperial City precincts. P_0 (Danei) is 15 squares wide and deep overall; P_1 (Dongge City) and P_2 (Xige City) are each 5 squares wide and 14 squares deep; P_3 (Dongjia City) and P_4 (Xijia City) are each 2.5 squares wide and 14 squares deep; P_5 (Xuanwu City) is 30 squares wide and 4 squares deep; P_6 (Yaoyi City) is 30 squares wide and 2 squares deep; P_7 (Yuanbi City) is 30 squares wide; P_8 (Imperial City) is 30 squares wide，and its depth from the south wall of the Danei occupies 9 squares; P_9 (Dong City) is 9 squares wide and nearly 21 squares deep; P_{10} (Hanjiacang City) is 9 squares wide. The grid scales of these precincts completely match the modular scales in **Table 6**，thus geometrically verifying once again that the 24 *zhang* (70.56 m) modular unit permeated the entire layout process of the Palace and Imperial City precinct.

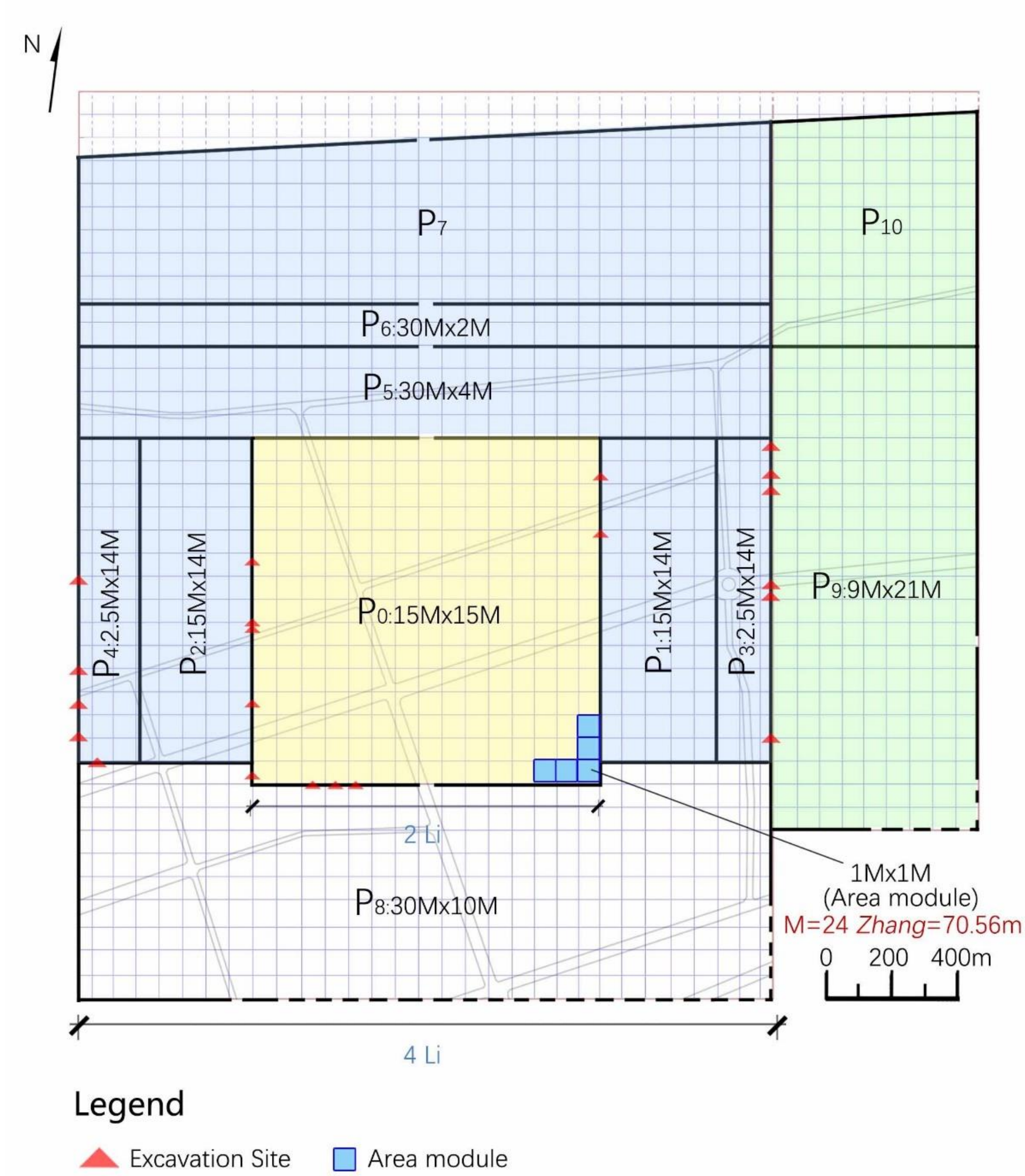

Fig.7: Grid Layout Analysis Diagram of Various precincts in the Palace /Imperial City

Relationship Between the Danei's Modular Unit and the Outer City(Including City Walls, Streets, and Wards)

The previous research, through the three stages of preliminary proportion calculation, verification of modular relationships, and arrangement of the modular grid, discovered that 24 *zhang* (70.56 m) was the fundamental module for the layout design of the Palace and Imperial Cities. Is this module applicable to the layout of the Outer City?

The Outer City of Luoyang had walls on all four sides, forming a basically square overall shape. The north wall runs slightly SW-NE and is slightly shorter than the south wall; the west wall is also slightly shorter than the east wall. The east and south walls are more regular and perpendicular to each other, so they are selected here to analyze the modular relationship with the Danei.

Table 7: Modular relationship table of the east wall and south wall of the Outer City wall (M=70.56 m)

Object Name	Surveyed Length(m)	Surveyed Module Multiple(M)	Design Module Multiple(M)	Module Multiple Match Rate(%)
Square Danei	1040	14.74M	15M (15×1)	98.3
Outer City East Wall	7312	103.63M	105M (15×7)	98.7
Outer City South Wall	7290	103.32M	105M (15×7)	98.4

From the calculation results in **Table 7**，the square Danei，the Outer City East Wall，and the Outer City South Wall all use 24 *zhang* (70.56m) as the modular unit. One Sui-Tang *li* = 180 *zhang* × 2.94m/*zhang* = 529.4m， so the Danei's width/depth of 1040m is approximately 2 *li*. The Outer City walls use the Danei's side length as an enlarged module，which is 7 times the Danei's module，i.e.，7M = 14 *li*. It is evident that when Yuwen Kai planned and designed Luoyang， although he used an enlarged modular unit for the large-scale Outer City layout， the 24 *zhang* (70.56m) unit remained the most fundamental module.

As shown in T**able 7**，the Outer City employed a modular layout method similar to the Palace and Imperial Cities，differing mainly in the size of the module used. As shown in **Fig.8**，if the Outer City is arranged using the Danei as the modular unit for gridding，the Outer City would be 7 squares wide and 7 squares deep，which matches the modular relationship calculated from the survey data.

As shown in **Fig. 8**，Dingdingmen Street，also known as the Tian Street，ran from the Duan Gate of the Palace/Imperial City in the north to the Dingding Gate in the south. It was the central north-south axis traversing the entire city of Luoyang and its most important street.The results calculated in **Table 8** indicate that the width of Dingdingmen Street is approximately 2M，and its depth is approximately 60M.

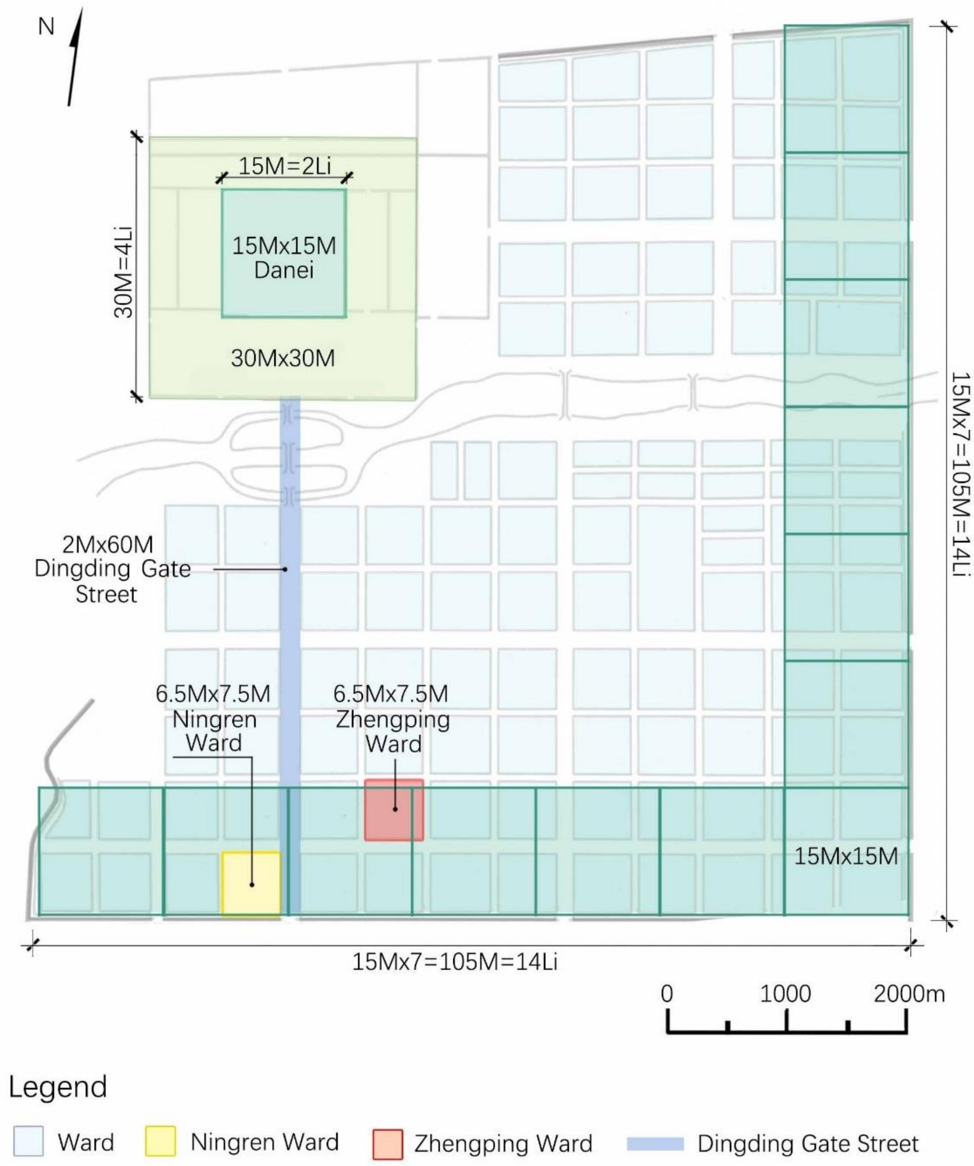

Fig.8: Diagram of Modular Relationship between Palace/Imperial City and Outer City (including wards and roads)

Table 8: Modular Relationship of Dingdingmen Street (M=70.56m)

Direction	**Surveyed Value(m)**	**Surveyed Module Multiple**	**Design Module Multiple**	**Module Multiple Match Rate(%)**
Width	141	1.99M	2M	99.9
Depth	4200	59.5M	60M	99.2

The second street west of Dingdingmen Street, located between Congzheng Ward and Ningren Ward, had a width of about 71 meters according to survey data, i.e., 1M. Starting in April 2020, the Institute of Archaeology, CASS, and the Luoyang Municipal Institute of Cultural Relics and Archaeology jointly conducted excavations at the Zhengping Ward site. The widths of the roads between Zhengping Ward and its adjacent wards to the east, west, and south were about 71 meters, approximately 1M.

Archaeological surveys have identified 12 north-south streets and 6 east-west streets in the ward precinct south of the Luo River; 4 north-south streets and 5 east-west streets were found in the ward precinct north of the Luo River. According to information publicly released by the Luoyang Tang City Archaeological Team, Institute of Archaeology, CASS, in September 2025, roads leading to city gates in Luoyang were generally about 71m wide (approx. 1M), while roads not leading to gates were generally about 34m wide (approx. 1/2 M), the latter being understood as a reduced module.

Sui-Tang Luoyang had numerous wards. Ningren Ward was the first ward to have its dimensions fully and accurately measured. It was the first ward west of Dingdingmen Street, bordering the Outer City's south wall to the south, Dingdingmen Street to the east, Yongtongmen Street to the north, and the second street west of Dingdingmen Street to the west. Ningren Ward's plan is a north-south oriented rectangle, with distances between ward walls being 527m north-south and 462m east-west.

Starting in April 2020, the Institute of Archaeology, CASS, and the Luoyang Municipal Institute of Cultural Relics and Archaeology jointly excavated the Zhengping Ward site. To date, the boundaries, precise location, road network structure, and the layout and positional relationships of large residential complexes within Zhengping Ward have been confirmed. Zhengping Ward measures 464.6 meters in width (east-west) and 533.6 meters in depth (north-south).

Table 9: Modular Relationship Table for Intra-City Wards (M=70.56m)

Ningren Ward

Direction	Surveyed Value(m)	Module Unit	Surveyed Module Multiple	Design Module Multiple	Module Multiple Match Rate(%)
Width	462	M	6.54M	6.5M	99.3
Depth	527	M	7.47M	7.5M	99.6

Zhengping Ward

Direction	Surveyed Value(m)	Module Unit	Surveyed Module Multiple	Design Module Multiple	Module Multiple Match Rate(%)
Width	464.6	M	6.58M	6.5M	98.8
Depth	533.6	M	7.56M	7.5M	99.2

As shown in **Table 9**, both Ningren Ward and Zhengping Ward measure 6.5M in width and 7.5M in depth. Moreover, as previously discussed, the use of 1/2M was also a standard practice in ancient China's modular system. Two halves of 1/2M combine to form 1M. It should be noted that the depth of 7.5M corresponds exactly to one Sui-Tang *li* (360 *bu* in the early Sui). 1 *li* = 1.47m/*bu* × 360 *bu* = 7.5 × 70.56m = 529.2m. As shown in **Fig.9**, the road between the east-west adjacent Ningren Ward and Congzheng Ward is 71m wide (approx. 1M), while Ningren Ward is 6.5M wide; added together they equal 7.5M, which is one Sui-Tang *li*. Thus, Ningren Ward and Zhengping Ward are truly "one-*li* wards."

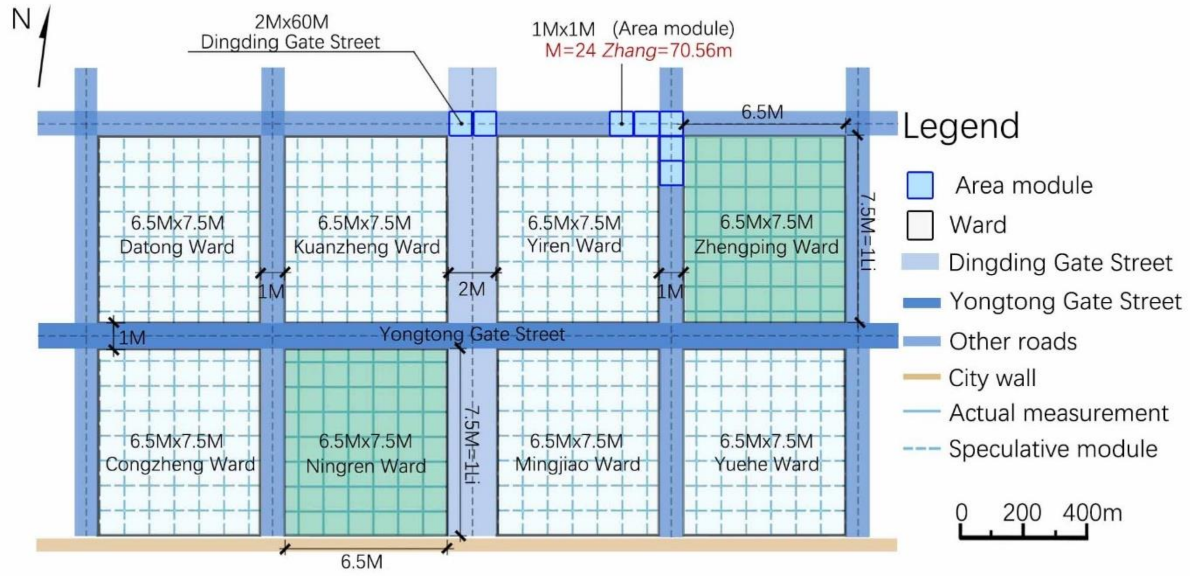

Fig.9: Diagram of Modular Relationships in the Ningren Ward - Dingdingmen Street - Zhengping Ward precinct (Drawn by the author)

Circle-Square Relationships and Symbolic Meaning in Compositional Proportions

Ancient Chinese capital layouts traditionally followed the principle of "Imitating Heaven and Modeling Earth", and Sui-Tang Luoyang was no exception. So how did the planner Yuwen Kai implement this tradition in the layout design of the Palace and Imperial Cities?

The answer lies within the compositional proportions of the several rectangles centered on the Danei. The essence of compositional proportions is number. The Western Han text *Zhoubi Suanjing* (Volume I, 《周髀算经》) records a dialogue between the Duke of Zhou（周公） and Shang Gao（商高）:

The Duke of Zhou ever addressed Shang Gao, saying, "I have privately heard, Sir, that you are skilled in numbers. May I ask, of old, Bao Xi（包牺）established the degrees of the circumference of heaven. Heaven cannot be ascended by steps, earth cannot be measured by a footrule – I ask, from where do the numbers arise?"Shang Gao replied, "The method of numbers comes from the circle and the square."[36] （pp.1）

"When the myriad things have their circuits complete, the circle and square are employed. When the great artisan creates institutions, the compass and square are established."[36]（pp.13）

Shang Gao's statement that the "method of numbers" "comes from the circle and the square" means that numbers originate from the construction of squares and circles, reflecting the ancient Chinese idea of the unity of number and form. Following this line of thought, we select some rectangles based on the Danei and use AutoCAD software to explore the square-circle construction relationships contained within the rectangles used in the Palace/Imperial City layout and their cultural symbolic meanings. Specific analysis is as follows:

As shown in **Fig.10**, P_0 (Danei) is 1040 meters square and can inscribe a circle of equal diameter. Also, the sum of the depths of P_6 (Yaoyi City), P_5 (Xuanwu City), P_0 (Danei), and P_8 (Imperial City) is 134 + 280 + 1040 + 655 = 2109 meters. The widths of P_6 (Yaoyi City) and P_8 (Imperial City) are both 2100 meters. 2109:2100 ≈ 1:1, meaning the combination of P_6, P_5, P_0, and P_8 forms a large square, whose precinct is four times that of the Danei, with a side length of approximately 30M. Furthermore, enclosing the entire Palace/Imperial City precinct (including the Dong City and Hanjiacang City) also yields a larger square with a match rate of 98.4%. Similarly, these two squares can each inscribe a large circle, forming the "Square-Inscribed Circle Diagram" (方圆图)found in the *Zhoubi Suanjing* and *Yingzao Fashi*（《营造法式》）.

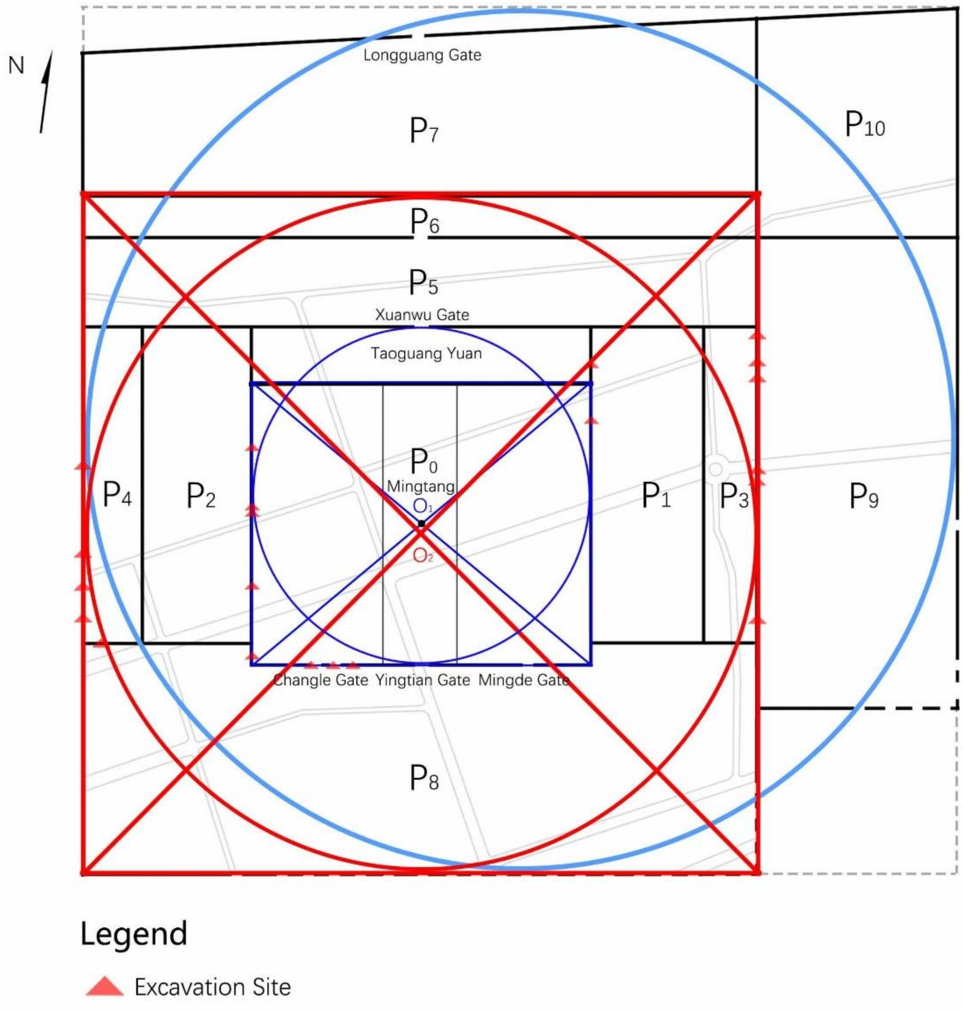

Fig.10: Analysis Diagram of Square-Circle Relationships Contained in 1:1 Rectangles

Connecting the diagonals of this large square yields an intersection point O_2. The distance from this point to the south wall of P_0 (Danei) is: 2100/2 – (725 – 70) = 395m. Meanwhile, connecting the diagonals of the three rectangular precincts (eastern, central, western) within the Danei yields an intersection point O_1. The depth of these three precincts within the Danei is 884m each, so O_1's distance from the south wall of P_0 is 884/2 = 442m. The Mingtang, located on the central axis of P_0, is 445m south of the Danei's south wall, differing from O_1 by only 3m, almost coinciding. This indicates that the Mingtang, as the most important ritual *bu*ilding

，was precisely located at the geometric center of the eastern，central，and western sections of the Danei. Determining the center point in Palace/Imperial City layout design is called "Establishing the Pivot" (立极).

Points O_1 and O_2 lie on the central axis of P_0. Calculating the distance between them: 445 – 395 = 50m. Restored to the actual scene，point O_2 would be located in the grand courtyard 50m south of the Mingtang，consistent with the spatial ritual tradition of the emperor residing in the central realm and addressing Heaven from the court.

As shown in **Fig.11**，using the two vertices on the south side of P_0 (Danei) as centers and the diagonal of P_0 as the radius，draw arcs northward. These arcs are precisely tangent to the partition wall between P_6 (Yaoyi City) and P_7 (Yuanbi City). The rectangle generated by this construction method is a "√2 rectangle，" whose short-to-long side ratio is 1:√2，convertible to the integer ratio 5:7.

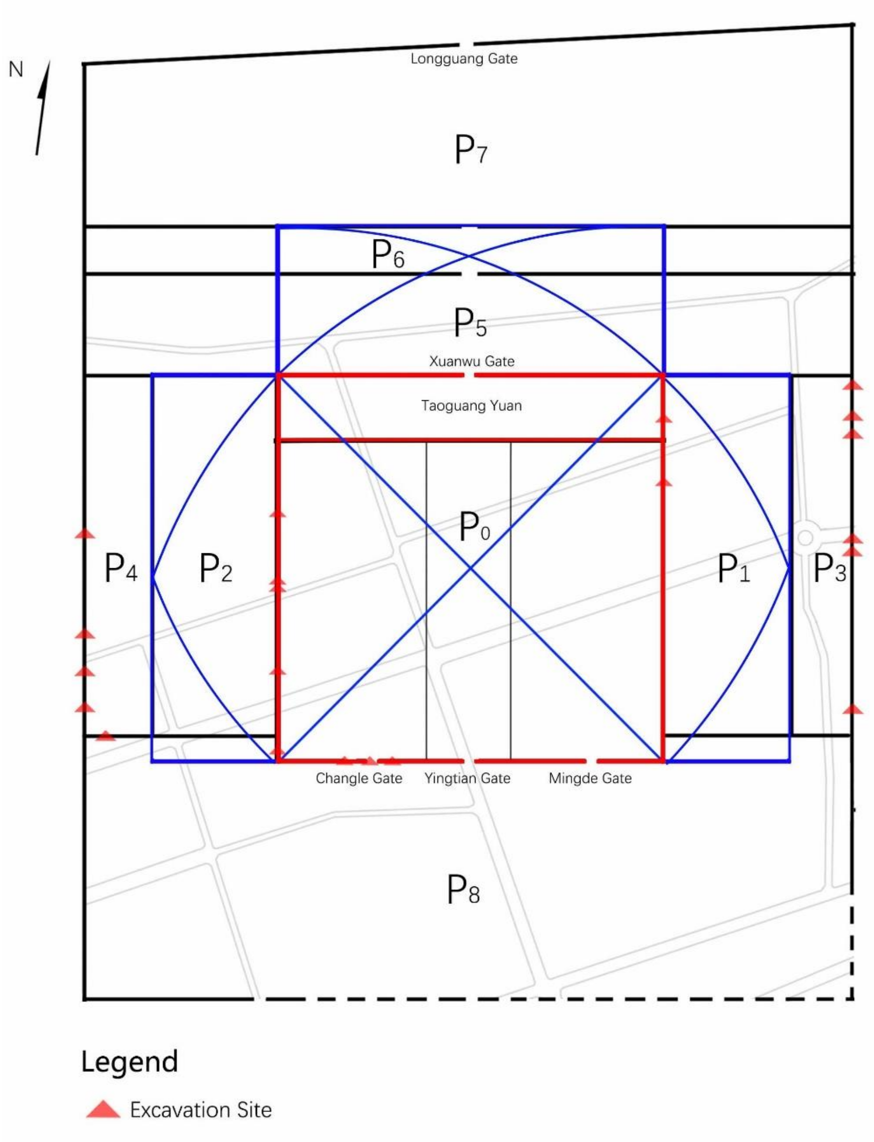

Fig.11: Analysis Diagram of Square-Circle Relationships Contained in 3:4 and 5:7 Rectangles

The sum of the north-south depths along the east wall of P_0 (Danei)，P_5 (Xuanwu City)

，and P_6 (Yaoyi City) is 1040 + 280 + 134 = 1454m. The east-west width of Xuanwu City and P_6 (Yaoyi City) is 2100m. Their ratio is 1454:2100 ≈ 5:7. Similarly，using the two vertices on the east side of the square as centers and the diagonal as the radius，drawing arcs westward would make them tangent to the west wall of P_5 and P_6，yielding another √2 rectangle.

As shown in **Fig.11**，using the two vertices on the west side of P_0 (Danei) as centers and the diagonal as the radius，draw arcs eastward. These arcs intersect precisely at the partition wall between P_1 (Dongge City) and P_3 (Dongjia City). The rectangle generated by this construction method has a long-to-short side ratio of √7/2 : 1，approximating the integer ratio 4:3. The author suggests that the occurrence of the 2.5M width in P_3 (Dongjia City) and P_4 (Xijia City) is largely a result of this proportion.

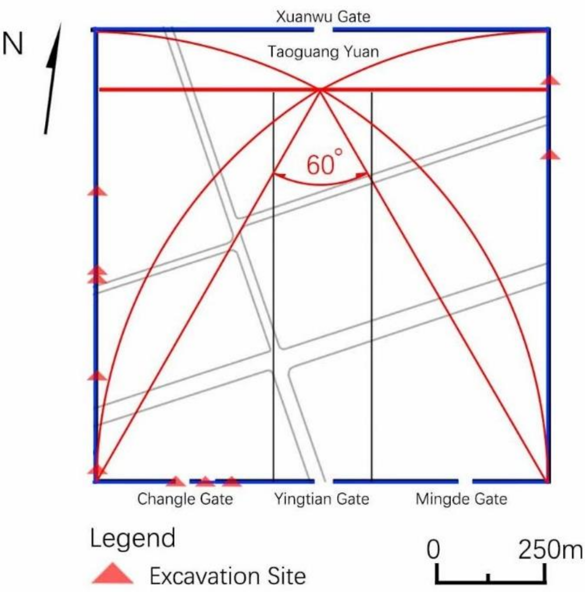

Fig.12:Analysis Diagram of Square-Circle Relationships Contained in 6:7 Rectangle

As shown in **Fig.12**，using the two vertices on the south side of P_0 (Danei) as centers and the side length of P_0 as the radius，draw arcs northward. These arcs intersect precisely at the midpoint of the southern corridor of the Taoguang Garden. The rectangle generated by this construction method has a short-to-long side ratio of √3/2 : 1，convertible to the integer ratio 6:7，which can inscribe an equilateral triangle.

The aforementioned square-circle construction relationships vividly demonstrate the method described in theZhoubi Suanjing: "Use the square as the standard，derive the circle from the square." Layout design was called "planning with compasses" (规画) in the Tang Dynasty，literally meaning "using compasses to draw circles." However，the circles in the Palace/Imperial City layout of Luoyang were invisible，hidden within the rectangles of different depth-width ratios centered on the Danei. One must delve into its deep structure to comprehend the ingenuity of Yuwen Kai's original layout design.

The ancients believed "the square pertains to Earth，the circle pertains to Heaven; Heaven is round，Earth is square."[36](pp.8) In the layout practice of the Palace/Imperial City，the square is visible，while the circle is not; it is only concealed within the compositional proportions of different rectangles. The ratios 5:7，6:7，3:5，and 3:4 discussed above are universally found in the layouts of ancient Chinese capitals，architectural complexes，and individual *bu*ildings，referred to by scholars as the "Harmony Ratios of Heaven and Earth."(天地之和比).This construction tradition can be traced back to the round mound(圆丘) and square mound（方丘） of the Hongshan Culture at Niuheliang，Liaoning，dating to 5000±130 years ago. The measured diameter ratios of the three rings of the round mound **(Fig.13)** are 11:15.6:22，extremely close to 1:√2:2. Scholar Feng Shi believes the three rings were not created directly as concentric circles of different radii but originated from square-circle constructions. As a place for worshipping Heaven, its layout naturally reflected the ancient unique understanding of the cosmos: first create an inner circle, then circumscribe a square around it, then create a circumscribed circle, then create an outer circumscribed square, and finally an outer circumscribed circle, thus completing the symbolism of

Fig.13:Remains of the Hongshan Culture Round Mound at Niuheliang, Liaoning Source: Feng Shi, *Civilization and Its Boundaries: Astronomy, Thought, and Institutions in Ancient China*

Heaven and Earth[37]. Differently, the symbolism of Heaven and Earth in the large-scale Palace/Imperial City of Sui-Tang Luoyang could not be directly achieved through square-circle construction but was instead hidden within specific proportions formed by the arrangement of modular units (**Fig.14**). In the Palace/Imperial City layout of Sui-Tang Luoyang, the planner Yuwen Kai used these compositional proportions precisely because they originally arose from square-circle constructions, and this square-circle relationship symbolizes the "Round Heaven, Square Earth"（天圆地方） cosmology.

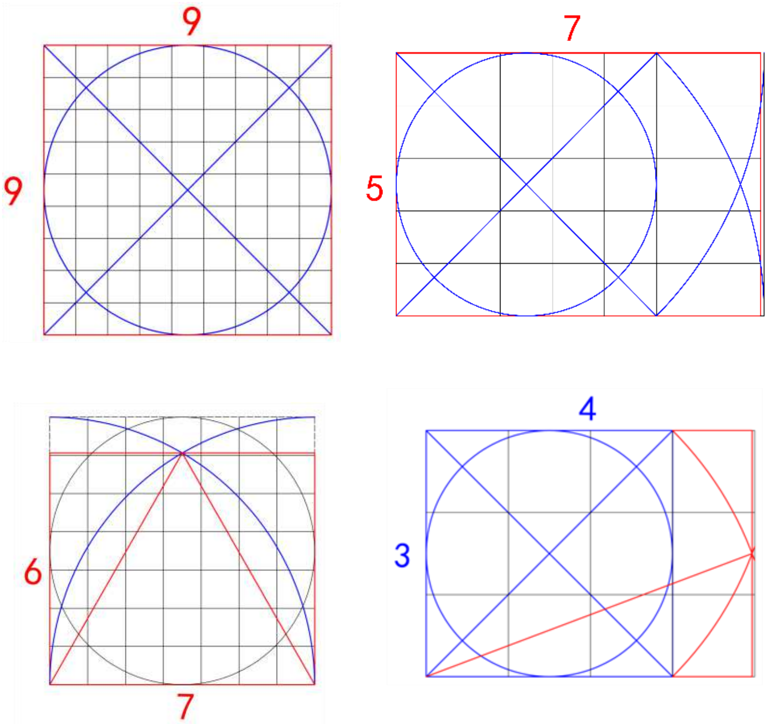

Fig.14: Diagram of Square-Circle Relationships Contained in the compositional ratios of 1:1, 5:7, 6:7, 3:4 (Drawn by the author)

Discussion

Drawing upon the research presented above, two principal findings have been established:

First, in response to the scale variations among different zones, the layout design of Sui-Tang Luoyang employed a three-tier modular system:the medium-scale Palace/Imperial City layout used 24 *zhang* (70.56m) as the fundamental module; the large-scale Outer City walls used 360 *zhang* (1058.4m) as the enlarged module; and the small-scale streets，wards，and Jiacheng used 12 *zhang* (35.28m) as the reduced module，also called the "half-unit." This fully reflects Yuwen Kai's planning wisdom of "defining form with numbers，controlling by hierarchy" in layout design. The fundamental module used in the Palace/Imperial City layout played a pivotal role connecting different scales: the enlarged module is 15 times it，and the reduced module is half of it. It connected the large，medium，and small-scale precincts of the entire capital into an organic whole with internal consistency，forming a complete spatial logic chain linking paths - streets - wards - Danei - Palace City - Imperial City - Outer City. This paper chose the modular layout principles of the Palace and Imperial Cities as the breakthrough point for finding the layout rules of the entire capital for this reason.Although only Ningren Ward and Zhengping Ward have been fully excavated and have publicly available accurate dimensions，archaeological survey work has shown that the wards of Luoyang were arranged as regularly as a chessboard. It is believed that more and more archaeological results will prove that other wards also conform to this modular layout principle.

Second, the adoption of specific compositional proportions containing inherent circle-square relationships in the layout of the Palace-Imperial Complex stems from the traditional capital-building philosophy of "imitating heaven and modeling earth." The application of specific proportions in ancient capital planning is not unique to China, and the significance of such compositional proportions varies greatly across different cultural spheres worldwide, owing to distinct cultural traditions that impart different symbolic meanings to spatial forms[38,39].Rather than simplistically treating certain proportions as spatial symbols of Chinese culture, this study employs mathematical and geometric methods to extract the compositional proportions—such as 1:1, 5:7, 6:7, and 3:4—used in the modular layout of Luoyang. It clarifies the relationship between these proportions and circle-square constructions while integrating insights from the humanities to reposition these proportions and numbers within the spatial traditions of ancient Chinese capital-building, thereby uncovering the underlying cosmological concept of "Round Heaven, Square Earth."

It should be noted that current archaeological work has primarily focused on Palace City, Imperial City, and the key areas of Outer City. Due to practical constraints, comprehensive excavation of the entire city is not feasible. The three-tier modular system identified in this study for the layout of Sui-Tang Luoyang is based on archaeological materials available prior to 2025. While archaeological surveys have indicated that the wards and streets in the Outer City are arranged as regularly as a chessboard, the possibility cannot be ruled out that future discoveries in specific areas of the Outer City may deviate from this modular framework.

The module deduction method based on compositional proportions presented in this study not only reveals the modular layout principles of Sui-Tang Luoyang but also offers a methodological model for deciphering the formative logic of other ancient capitals. Compared to earlier approaches reliant on expert-drawn diagrams based on personal experience, the "three-step analytical method" introduced here — beginning with the calculation of compositional proportions, proceeding to the deduction of modular relationships, and concluding with verification through grid arrangement—undoubtedly enhances the scientific rigor and reliability of the research. Preliminary calculations by the author based on archaeological survey data and maps suggest that Sui-Tang Chang'an, which was constructed using the Daxing Palace as a reference module, also employed a fundamental module of 24 *zhang* (70.56 m) for the entire capital. Using core areas such as the Great Interior or the palace city as reference units for grid-based layout design is a construction tradition shared by ancient capitals across East Asia, including China, Japan, and Korea[40,41]. From this perspective, the module deduction method based on compositional proportions proposed in this study can provide a methodological reference for uncovering the shaping principles behind similar urban forms in East Asia.

Acknowledgements
Not applicable.

Author contributions
SWT calculated the archaeological data and drafted the manuscript; CYJ and WXY produced Figures 1-4 and 7-12; GZZ assisted in calculating the archaeological data; ZYH was responsible for project organization and funding acquisition;All authors read and approved the final manuscript.

Funding
This work was supported by: National Office for Philosophy and Social Sciences [No.23ZD11]，National Ethnic Affairs Commission of the People's Republic of China[No.2024GMD006].

Availability of data and materials
All data supporting the findings of this study are available within the paper .

Declarations

Competing interests
The authors declare no competing interests.

References

1. Addis, F. *Rome: Eternal City.* (Head of Zeus, 2018).
2. Liu, C. et al. The urban form of ancient Chinese capital cities. *Urban Morphology.* **26,** 154–172 (2022).
3. Steinhardt, N. S. *Chinese Imperial City Planning.* (University of Hawai'i Press, 1990).
4. Schinz, A. *The Magic Square: Cities in Ancient China.* (Axel Menges, 1996).
5. Boyd, A. *Chinese Architecture and Town Planning 1500 B.C.–A.D. 1911.* (University of Chicago Press, 1962).
6. Chang, S. D. Some observations on the morphology of Chinese walled cities. *Annals of the Association of American Geographers* **60,** 63–91 (1970).
7. James, S. The city walls of Han Ch'ang-an. *T'oung Pao.* **64,** 1–46 (1978).

8. Qin, Z. et al. Urban archaeology in Kaifeng, a capital city of dynastic China: progress and insights. *World Archaeol.* **54,** 680–699 (2022).
9. Wu, T. H. *Cangchun: The Planning of Yuan Dadu.* (Commercial Press, 2024).
10. Sit, V. F. S. *Beijing: The Nature and Planning of a Chinese Capital City.* (Wiley, 1995).
11. Heng, C. K. *Cities of Aristocrats and Bureaucrats: The Development of Medieval Chinese Cityscapes.* (University of Hawai'i Press, 1999).
12. Xiong, V. C. *Sui-Tang Chang'an: A Study in the Urban History of Late Medieval China.* (University of Michigan Press, 2000).
13. Yin, L. R., Wang, T. & Adeyeye, K. A comparative study of urban spatial characteristics of the capitals of Tang and Song dynasties based on space syntax. *Urban Sci.* **5,** 34 (2021).
14. Shuji, B. The model of ancient Chinese capitals—measurement systems in urban planning. *Proceedings of the Japan Academy* **93,** 724–745 (2017).
15. Senoo, T. *Chang'an in the Sui and Tang Dynasties and the Comparative History of East Asian Capitals* (trans. Gao, B. & Guo, X.) (Northwest University Press, 2019)
16. Chen, S. L., Dong, Y., Chen, X. Y., Xu, X. Y., & Gong, J. B. Analysis of the Characteristics and Ideas of Ancient Urban Land-Use Based on GIS and an Algorithm: A Case Study of Chang'an City in the Sui and Tang Dynasties. *Remote Sens.* **15,** 2962 (2023).
17. Kiang, H. C. Imperial cities: critical changes in urban paradigm from Sui-Tang to Song. in *Routledge Handbook of Chinese Architecture* 70–86 (Routledge, 2022).
18. Fu, Z. H. Urban roads: inheritance and development of traditional Chinese urban planning wisdom. in *A Brief History of Transportation in China* (ed. Lu, H.) 165–196 (Springer, 2025).
19. Ho, P. T. Lo-yang, A.D. 495–534: a study of physical and socio-economic planning of a metropolitan area. *Harvard Journal of Asiatic Studies* **26,** 52–101 (1966).
20. Wang, R. K. & Hein, C. From natural environment to artificial system: Chang'an and its water system in the Western Han dynasty. *Frontiers of Architectural Research* **11,** 440–452 (2022).
21. Chen, S. L. et al. New archaeological discoveries based on spatial information technology and cultural analysis: taking the study of the spatial relationship between ancient Chinese capitals and the natural environment as an example. *Remote Sens.* **14**, 3298 (2022).
22. Bai, Q. T., Lyu, H. Y. & Gao, M. C. Spatiotemporal evolution of capitals and settlements in the Luoyang Basin from the Xia to Han-Wei periods. *npj Herit. Sci.* **14,** 5 (2026).
23. Zhao, H. J. What matters in the choice of a national capital location in Chinese history: an empirical analysis based on quantitative economic history. *Spatial Economic Analysis* **16,** 355–377 (2021).
24. Wheatley, P. *The Pivot of the Four Quarters: A Preliminary Enquiry into the Origins and Character of the Ancient Chinese City.* (Edinburgh University Press, 1971).
25. Wright, A. F. *The cosmology of the Chinese city. in The City in Late Imperial China* (ed. Skinner, G. W.) 33–73 (Stanford University Press, 1977).
26. Wu, T. H., Xu, B. & Wang, X. R. How ancient Chinese constellations are applied in the city planning? An example on the planning principles employed in Xianyang, the capital city of Qin dynasty. *Sci. Bull.* **61,** 1634–1636 (2016).
27. Fu, X. N. *Research on the Planning Methods of Ancient Chinese Urban Planning, Architectural Group Layouts, and Architectural Design* (2nd edn.) (China Architecture & Building Press, 2015).
28. Wang, S. S. An analysis of the planning techniques of Chang'an City in the Sui and Tang dynasties. *City Planning Review* **33,** 55–58 (2009).
29. Fan, J. & Gu, T. C. Comparative analysis of spatial patterns and driving factors of historic districts in the Chinese four ancient capitals. *npj Herit. Sci.* **13,** 245 (2025).
30. Kang, D. X., Cheng, J. L., Yin, J. & Li, H. H. What role does "great heritage site"

conservation play in shaping sustainable urban intensity patterns? An empirical study from Luoyang. *Sustainability.* **17,** 3882 (2025).
31. Xiong, V. C. *Capital Cities and Urban Form in Pre-modern China: Luoyang, 1038 B.C. to 938 A.D.* (Routledge, 2016).
32. Li, S. L. Luoyang: the ancient capital with a millenary history. in *Development History of the Grand Canal Cities* **2,** 305-332 (Palgrave Macmillan, 2025).
33. Xu, S. *Shuowen Jiezi* (Proofread by Xu, X.) (Zhonghua Book Company, 1963).
34. Liu, S. C. *Annotated Translation of Gongyang Commentary on the Spring and Autumn Annals: The First Year of* Duke Yin. (Zhonghua Book Company, 2010).
35. Wei, Z. & Linghu, D. F. *Book of Sui: Biography of Yuwen Kai.* 1593 (Zhonghua Book Company, 1973).
36. Cheng, Z. Y. & Wen, R. J. *Annotated Translation of Zhoubi Suanjing* **1,** (Shanghai Ancient Books Publishing House, 2012).
37. Feng, S. *Astronomy and Humanities in Ancient China* (Revised edn.) 293-311 (China Social Sciences Press, 2006).
38. Jiang, H. & Zhou, Y. Evolution of the “Golden Royal City”: analytical study on urban forms of ancient capital cities in Myanmar. *Frontiers of Architectural Research* **12,** 714–731 (2023).
39. Maroufi, H. Urban planning in ancient cities of Iran: understanding the meaning of urban form in the Sasanian city of Ardašīr-Xwarrah. *Planning Perspectives* **35,** 1055–1080 (2020).
40. Niu, R. Z. *The Origin of East Asian Medieval Capital Construction System: The Ancient City of Ye* (1st ed.) (Routledge, 2021).
41. Wang, Z. S. On the influence of the Lindedian Pavilion, Daminggong Palace, Tang Chang’an City, upon the design of palaces in the Japanese capitals Heijokyo and Heiankyo. *Chinese Archaeology* **2,** 78–83 (2002).